Proceedings of the 17th IASTED International Conference
MODELLING AND SIMULATION
May 24-16, 2006, Montreal, PQ, Canada

ESTIMATION OF VELOCITY PROFILES IN NATURAL CHANNELS DURING HIGH FLOODS

ALESSIO BURNELLI, TOMMASO MORAMARCO
National Research Council, IRPI
Via M. Alta 126, 06128 Perugia, Italy
e-mail A.Burnelli@irpi.cnr.it, T.Moramarco@irpi.cnr.it

CARLA SALTALIPPI
Dept. of Civil and Environmental Engineering, University of Perugia
Via G. Duranti 93, 06125 Perugia, Italy
e-mail saltalip@unipg.it

ABSTRACT
Four different approaches to describe the velocity profile along verticals in natural channels have been investigated. Three methods consider the classical logarithmic law with additional terms, which introduce the dip-phenomenon and the curvature of the velocity profile. The latter uses a modified entropic distribution of velocity profiles. Methods have been tested to reproduce the velocity profile during high flows, when velocity measurements can be carried out only in the upper portion of the flow area. An equipped hydrometric site located along the Po river, in northern Italy, has been used as case study. Four velocity measurements have been selected and the hypothesis of high flood has been considered. The methods reliability has been investigated in terms of percentage errors in estimating both the mean velocity along each sampled vertical and the mean flow velocity. It was found that the entropic approach performed better than the other ones, showing that it can be efficiently applied for high flood conditions.

KEY WORDS
Velocity measurements, Rating curve, Floods.

1. Introduction

A quick and accurate determination of flow passing through a river section is very important for a large number of engineering applications such as flow forecasting model and real time water resources management. Rating curve knowledge is fundamental to this aim and it can be obtained from mean flow velocity, which has to be estimated, usually on the basis of velocity measurements sampled in the flow area. The rating curve accuracy is strictly connected to experimental data availability which have to be referred both to low and high flow depths ([1]). Sampling procedure of velocity measurements in a river cross section during high floods could be difficult and particularly dangerous, mainly in the lower portion of the flow area. On the other hand the value of maximum flow velocity could be more easily obtained since its position is located in the upper portion of the flow area where velocity measurements can be carried out also during high flow conditions ([2]). Many studies were addressed to investigate the spatial velocity distribution ([3], [4], [5]) and the entropic linear relationship between the mean and maximum velocity, represents an useful and efficient method to estimate the flow rate during high flood ([5], [6]). This procedure has been investigated and applied to different rivers sites located in America and Europe ([7], [8]) furnishing interesting results, even though in some cases the estimation of mean flow velocity was not accurate (absolute percentage errors greater than 30%) ([9]).

The possibility to reduce errors in mean velocity estimation has been also analyzed by reproducing the entire flow velocity characterization of events occurring in a river cross section. Entropic approaches and traditional methods, based on logarithmic behavior of velocity distribution along a vertical of the cross sectional area, have been developed. A first analytical entropic characterization of velocity profiles was introduced by Chiu ([5], [10]). This method was found to be onerous for practical applications since six parameters have to be estimated, besides M, and not sufficiently accurate in describing the actual behavior of velocity profiles close to sidewalls ([11]). Based on this model, further entropic approaches have been developed allowing to reduce the model complexity and errors in mean flow velocity estimation ([6], [11]).

Traditional logarithmic approaches describe velocity profiles by using equations with a limited number of parameters which can be determined on the basis of flow velocity measurements along each vertical. In particular, these approaches need a number of velocity measurements equal or greater than the parameters involved, along with the position of the velocity points sampled. Fenton ([12]) introduced a modified procedure of the traditional three-points or four-points method to

estimate mean velocity along a vertical. In fact, for the proposed procedure velocity sampling has not to be performed at fixed heights in the vertical from the bottom. The procedure has not been already used to describe natural channel flow. Another interesting approach was developed by Yang et al., who introduced a dip-correction factor to account the velocity dip phenomenon that always exists close to sidewalls ([13]). This approach has been tested only in smooth rectangular open channel flows.
The objective of the paper is to test the reliability of the aforementioned approaches to estimate the mean flow velocity in a natural river section during high flow, when sampling of velocity points is possible only in the upper portion of the flow area. The velocity data collected during four flood events at Pontelagoscuro site, along river Po, are used for the analysis.

2. Velocity profile distribution models

The classical logarithmic law describing velocity distribution along a vertical of a cross-sectional flow area, for turbolent flow over a rough bed, can be expressed as:

$$u(y) = \frac{u_*}{k} \ln \frac{y}{y_0} \tag{1}$$

where:
- y is the distance from the bottom;
- u_* is the shear velocity, $u_* = (gRS)^{0.5}$ (g is the gravitation acceleration, R is the hydraulic radius and S is the energy slope);
- k is the Karman constant;
- y_0 is the location where the velocity hypothetically equals zero.

To introduce the possibility that the velocity profile is deviating from a logarithmic form and that it may present a maximum value at some point under the water surface an additional term can be added to Eq. (1).
Fenton proposed the following expression ([12]):

$$u(y) = \frac{u_*}{k} \ln \frac{y}{y_0} + a_1 \frac{y}{D} \tag{2}$$

where a_1 is a further unknown coefficient, having the same units of velocity, and D is the vertical depth. If three velocity points, u_1, u_2 and u_3, are sampled at different positions y_1, y_2 and y_3, all the three unknown quantities included in Eq. (2), u_*/k, y_0 and a_1, can be estimated by calibration procedure.
Eq. (2) can be integrated to obtain the mean flow velocity value along the vertical:

$$u_v = \int_0^D u(y)dy = \frac{u_*}{k}\left(\ln\frac{D}{y_0} - 1\right) + \frac{a_1}{2} \tag{3}$$

Fenton introduced an additional quadratic term in Eq. (2) to better reproduce the curvature of velocity profile, yielding:

$$u(y) = \frac{u_*}{k} \ln \frac{y}{y_0} + a_1 \frac{y}{D} + a_2 \left(\frac{y}{D}\right)^2 \tag{4}$$

where a_2 is the additional unknown coefficient to be found by measurements. In this latter case four velocity points sampled at different positions along each vertical are needed. Therefore, the mean flow velocity can be derived as:

$$u_v = \int_0^D u(y)dy = \frac{u_*}{k}\left(\ln\frac{D}{y_0} - 1\right) + \frac{a_1}{2} + \frac{a_2}{3} \tag{5}$$

Yang et al. (2004) proposed a dip-modified logarithmic law for the velocity distribution in smooth uniform open channel flow, and it is based on two logarithmic depths, one from the bed, $\ln(y/y_0)$, and the other from the water surface, $\ln(1-y/D)$, [13]:

$$u(y) = u_*\left[\frac{1}{k}\ln\frac{y}{y_0} + \frac{\alpha}{k}\ln\left(1 - \frac{y}{D}\right)\right] \tag{6}$$

α is the dip-correction factor, depending only on the relative distance of the maximum velocity location, y_{max}, to the water depth, D. α can be estimated by Eq. (6) equating du/dy to 0, obtaining:

$$\alpha = \frac{D - y_{max}}{y_{max}} \tag{7}$$

Three velocity points, sampled at different distances from the bed, are needed to describe by Eq. (6) the entire velocity profile along the vertical.
The entropic model proposed by Moramarco et al. (2004) allows the estimation of the velocity profile using a simplification of the analytical formulation introduced by Chiu ([2], [10], [11]):

$$u(y) = \frac{u_{max_v}}{M}\ln\left(1 + \left(e^M - 1\right)\frac{y}{D-h}\exp\left(1 - \frac{y}{D-h}\right)\right) \tag{8}$$

where u_{max_v} is the maximum velocity sampled along the investigated vertical and h is the location of the maximum velocity in terms of distance from water surface.
M is the entropic parameter, which is a characteristics of the river cross section and can be estimated by using the linear relationship [10]:

$$u_m = \Phi(M)u_{max} \tag{9}$$

where u_m and u_{max} are the mean and the maximum flow velocity, respectively. Φ(M) can be expressed by [10]:

$$\Phi(M) = \frac{u_m}{u_{max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} \quad (10)$$

The entropic parameter M can be estimated, for the investigated cross section, on basis of pairs (u_m, u_{max}) of available data from measurements sampling ([7], [11]). Once M has estimated and u_{max_v} sampled, Eq.(8) furnishes the velocity profile along the vertical.

3. Case study

The four velocity distribution equations (Eq. (2), (4), (6) and (8)) were tested with the velocity data collected during four flood events at the river section of Pontelagoscuro along the Po river (Northern Italy). The total number of investigated verticals is 52.

Table 1 summarizes the main characteristics of the selected flood events. For each event the experimental data consist in:

1) velocity point measurements in different positions along each vertical;
2) distance of each vertical from left sidewall;
3) water stage;
4) mean velocity;
5) discharge.

Tab. 1. Stage, flow area (Area), mean velocity, u_m, maximum velocity, u_{max}, and discharge, Q, for the selected events.

Event	*Stage* (m)	*Area* (m^2)	u_m (ms^{-1})	u_{max} (ms^{-1})	*Q* (m^3s^{-1})
February 13, 1985	5.53	2052	1.13	1.80	2358
February 24, 1987	4.65	1853	0.94	1.43	1779
October 16, 1987	8.68	2448	2.04	2.71	5026
July 5, 1988	5.54	2105	1.07	1.59	2283

To simulate the sampling conditions during high flow, only measurements carried out in the upper portion of the flow area are considered to estimate the parameters included in all the four relations.

For the equipped site of Pontelagoscuro the entropic parameter, M, was estimated, on basis of 48 flow measurements performed during the period 1984-1992.

Φ(M) was found equal to 0.668 (see Fig.1) and, then, by Eq. (10), M=2.162. It is shown as the linear relationship underestimates the actual values of the mean flow velocity, mainly when the maximum velocity is greater than 2.0 ms^{-1}.

Unknown parameters of Eq. (2) and (6) need three points of measurements along each vertical to be estimated; the ones of Eq. (4) need four points and the application of Eq. (8) requires the value of u_{max_v} for each vertical so it seems that only one point is needed. Unfortunately, the localization of the maximum velocity u_{max_v} under the water surface is unknown, so this aspect has to be taken into account. For the case study here reported, and in general for wide channels, the position is not so deep below the water surface with the consequence of a very limited number of velocity points measurements needed for its estimation.

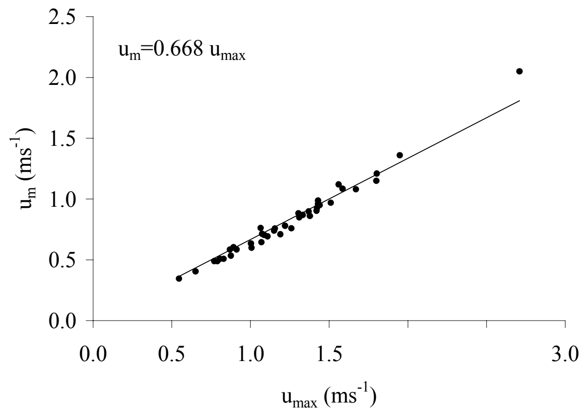

Fig. 1. Relation between mean and maximum velocities at the gauged river section of Pontelagoscuro.

The two velocity profiles of Figure 2 represent two typical and opposite situations: the first one corresponding to a wrong velocity profile estimation for almost all approaches – case a) – and the second one producing a small error in estimating the mean velocity value along the vertical for all the investigated methods – case b).

For each vertical the accuracy of the velocity distribution model was investigated by evaluating the percentage error in estimating the mean value of velocity, u_v.

Figure 3 shows the errors distribution, in magnitude, related to the dimensionless distance, x', defined as $x'=x/x_{SX}$ or $x'=x/x_{DX}$, with x representing the horizontal distance (positive or negative) of the considered vertical from the one in which u_{max} was observed (x=0); x_{SX} and x_{DX} is the distance of the vertical at x=0, from the left and the right sidewall, respectively.

Mean value of the errors, in magnitude, is 20.7% for Eq. (2), 21.7% for Eq. (4), 11.0% for Eq (6) and 6.3% for Eq. (8).

The cumulated frequency of the errors is shown in Figure 4. Eq. (2) and (4) produced errors greater than 10% for 63% and 56% of the investigated verticals, respectively. The error is greater than 10% for 48% and 13% of the verticals for Eq. (6) and (8), respectively.

These errors influence also the errors in estimating mean flow velocity values, which have been obtained by using the well-known velocity area method [14]. Table 2 shows that Eq. (2), (4) and (6) generally estimated a value of u_m less than the actual one. Eq. (8) performed better reducing significantly the percentage error values. Results obtained by applying the linear relationship, Eq. (9), have also shown. In this case, greater errors are associated to higher values of u_m (and consequently of u_{max}); whereas Eq. (8), which requires also the knowledge of the entropic parameter M, derived from the same linear relationship, is able to greatly reduce the errors.

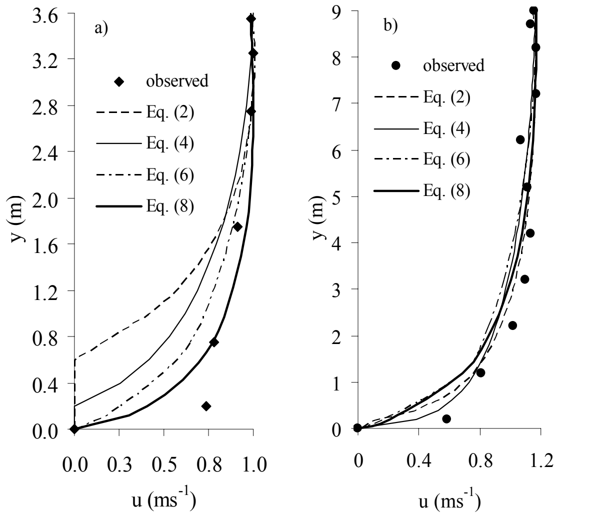

Fig. 2. Velocity profiles reproduced by the four investigated method: a) velocity profile at x= -134 m during the event of February 13, 1985; b) velocity profile at x= -55 m during the event of July 5, 1988. Distance x represents the horizontal distance of considered vertical from the one in which u_{max} was observed (x=0).

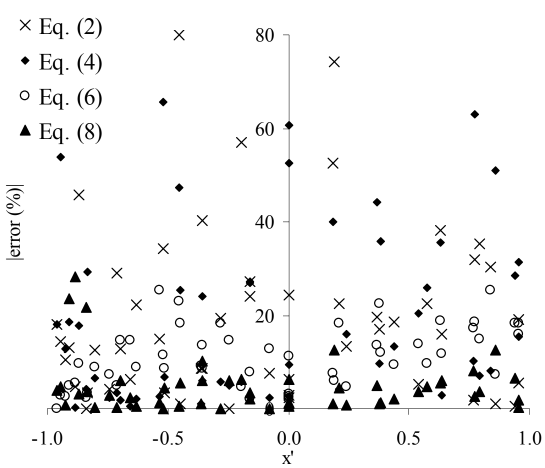

Fig. 3. Percentage error, in magnitude, in estimating the mean velocity along the 52 investigated verticals. Distance x' represents the adimensional horizontal distance considered vertical from the one in which u_{max} was observed (x=0).

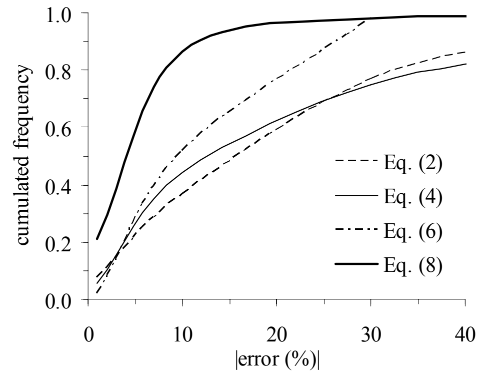

Fig. 4. Cumulated frequency of the percentage error, in magnitude, in estimating the mean velocity along the 52 investigated verticals.

Table 2. Percentage error in estimating the mean flow velocity for the selected events; high flow conditions have been surmised.

event	u_m	error (%)				
	(ms^{-1})	eq. (2)	eq. (4)	eq. (6)	eq. (8)	eq. (9)
February 13, 1985	1.13	-22.9	-15.5	-7.2	2.6	6.4
February 24, 1987	0.94	-22.0	-8.5	-11.6	-1.0	0.9
October 16, 1987	2.04	6.3	1.7	14.9	-2.7	-11.3
July 5, 1988	1.07	-17.4	-3.5	-6.3	-1.3	-1.4

4. Conclusion

The reliability analysis of different approaches in estimating the mean flow velocity, showed that the logarithmic methods produced, along each vertical, percentage errors greater than the ones corresponding to the application of the entropic approach. In terms of mean flow velocity, the entropic method was found more accurate also surmising the case of high flood. Based on these results, the entropic approach can be conveniently adopted to finalize the velocity measurements during high flood. In this way, the safety of personnel is guaranteed and the duration of the measurement is also significantly shorted.

Acknowledgements

The authors wish to thank the Enel of Mestre for providing the correntometric data used in this study. The authors are particularly grateful to Eng. Giorgio Galeati who spent part of his time to prepare a copy of the data. This work was partly funded by the National Research Council of Italy, under the Special Project GNDCI, with the financial support of the Agenzia Interregionale per il Fiume Po.

References

[1] S. Barbetta, F. Melone & T. Moramarco, Sull'accuratezza dei metodi di stima delle scale di deflusso, *Proc. of XXVIII National Conference on Hydraulics and Hydraulic Structures*, Potenza, September 16-19, 2002, Vol. I, 149-157.

[2] C.L. Chiu, Entropy and probability concepts in hydraulics, *Journal of Hydraulic Engineering, 113(5)*, 1987, 583-600.

[3] S.N. Ghosh &N. Roy, Boundary shear distribution in open channel flow, *Journal of the Hydraulic Division, ASCE, 96*(HY4), 1970, 967-994.

[4] S. Sulzer, P. Rutschmann & W. Kinzelbach, Flood discharge prediction using two-dimensional inverse modeling, *Journal of Hydraulic Engineering, ASCE, 128*(1), 2002, 46-54.

[5] C.L. Chiu, Entropy and 2-D velocity distribution in open channels, *Journal of Hydraulic Engineering, 114(7)*, 1988, 738-756.

[6] C.L. Chiu & C.A.A. Said, Maximum and mean velocities and entropy in open-channel flow, *Journal of Hydraulic Engineering, ASCE, 121*(1), 1995, 26-35.

[7] R. Xia, Relation between mean and maximum velocities in a natural river, *Journal of Hydraulic Engineering, 123(8)*, 1997, 720-723.

[8] T. Moramarco & C. Saltalippi, Entropic characterization of the mean flow velocity in natural channels, *Proceedings of the IASTED International Conference "Environmental Modelling and Simulation"*, St. Thomas, US Virgin Islands, November 22-24, 2004, 175-179.

[9] T. Moramarco & C. Saltalippi, Stima della velocità media in sezioni di un corso d'acqua naturale, *Proc. of XXVIII National Conference on Hydraulics and Hydraulic Structures*, Potenza, September 16-19, 2002, Vol. III, 263-274.

[10] C.L. Chiu, Velocity distribution in open channel flow, *Journal of Hydraulic Engineering, 115(5)*, 1989, 576-594.

[11] T. Moramarco, C. Saltalippi & V.P. Singh, Estimation of mean velocity in natural channels based on Chiu's velocity distribution equation, *Journal of Hydrologic Engineering, 9(1)*, 2004, 42–50.

[12] J.D. Fenton, The application of numerical methods and mathematics to hydrography, *Proc. 11th Australasian Hydrographic Conference*, Sydney 3-6 July 2002.

[13] S.Q. Yang, S.K. Tan & S.Y. Lim, Velocity distribution and dip-phenomenon in smooth uniform open channel flows, *Journal of Hydraulic Engineering, 130(12)*, 2004, 1179-1186.

[14] R.W. Herschy, *Streamflow measurements* (Elsevier, London, 1985).